



PHYS 101 – General Physics I Midterm Exam

Duration: 90 minutes

Saturday, 19 October 2024; 09:30

1. Two runners, A and B, are competing in a 100-meter race. They start at the same time. Runner A accelerates at a rate of 4 m/s^2 for 2 seconds, while Runner B accelerates at a rate of 3 m/s^2 for 3 seconds. After their initial acceleration periods, both runners continue to run at their maximum speeds until they finish the race.

(a) (15 Pts.) What is the time difference between A and B when the winner finishes the race? Who is the winner?

(b) (15 Pts.) How far behind is the runner-up when the winner finishes the race?

Solution: (a) During the acceleration period runner A covers the distance

$$x_A(t) = \frac{1}{2} a_A t^2 \rightarrow x_A(t) = (2 \text{ m/s}^2) t^2, \quad 0 < t < 2 \text{ (s)} \rightarrow x_A(2) = 8 \text{ m}$$

and reaches the maximum speed

$$v_A(t) = a_A t \rightarrow v_A(t) = (4 \text{ m/s}^2) t, \quad 0 < t < 2 \text{ (s)} \rightarrow v_A(2) = 8 \text{ m/s},$$

while runner B covers the distance

$$x_B(t) = \frac{1}{2} a_B t^2 \rightarrow x_B(t) = \left(\frac{3}{2} \text{ m/s}^2\right) t^2, \quad 0 < t < 3 \text{ (s)} \rightarrow x_B(3) = \frac{27}{2} \text{ m} = 13.5 \text{ m}$$

and reaches the maximum speed

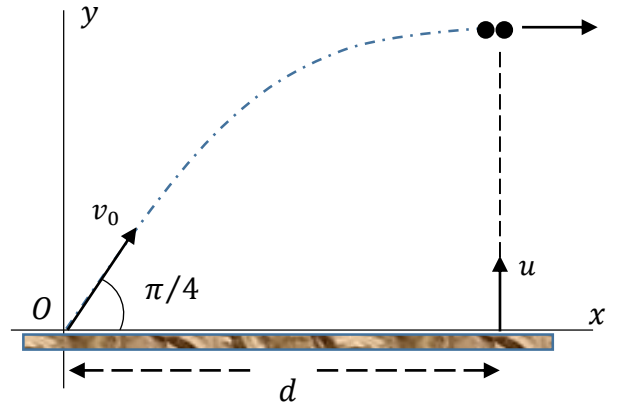
$$v_B(t) = a_B t \rightarrow v_B(t) = (3 \text{ m/s}^2) t, \quad 0 < t < 3 \text{ (s)} \rightarrow v_B(3) = 9 \text{ m/s}.$$

Clearly, the winner is B because of higher speed and less distance to cover to finish the race. Runner A covers the remaining 92 m in $92/8 = 11.5$ seconds, so total time of A is 13.5 seconds. Runner B covers the remaining 86.5 m in $86.5/9 = 9.61$ seconds, so the total time of B is 12.6 seconds. So the time difference is $\Delta t = 0.9$ s.

(b) The winner, Runner B finishes the race in 12.61 seconds. For $t > 2$ s the position of runner A is given by $x_A = (8 \text{ m}) + (8 \text{ m/s})(t - 2)$.

Therefore, when $t = 12.61$ s, the position of runner A is $x_A(12.6) = (8 \text{ m}) + (8 \text{ m/s})(10.61 \text{ s}) = 92.9 \text{ m}$. So runner A is $100 - 92.9 = 7.1$ m behind runner B when the race ends.

2. A target is thrown directly upward with speed u . A shooter is at a distance d away from the target's initial position, and fires a projectile at an angle $\pi/4$ with speed v_0 a time T after the target is launched. The projectile hits the target with a horizontal velocity as shown in the figure. Gravitational acceleration is g .
(Use $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$.)



- (a) (5 Pts.) Using the coordinate system given in the figure and assuming that the projectile is fired at time $t = 0$, write the position vectors for the projectile and the target.
 (b) (10 Pts.) Find v_0 in terms of d, u and g .
 (c) (10 Pts.) Find T in terms of d, u and g .
 (d) (5 Pts.) For which values of d, g , and u is the above scenario impossible?

Solution: (a)

$$\vec{r}_P = [v_0 \cos(\pi/4) t] \hat{i} + \left[v_0 \sin(\pi/4) t - \frac{1}{2} g t^2 \right] \hat{j} \rightarrow \vec{r}_P = \left[\frac{\sqrt{2} v_0 t}{2} \right] \hat{i} + \left[\frac{\sqrt{2} v_0 t}{2} - \frac{1}{2} g t^2 \right] \hat{j}$$

$$\vec{r}_T = [d] \hat{i} + \left[u(t + T) - \frac{1}{2} g(t + T)^2 \right] \hat{j}$$

(b) The projectile hits the target with a horizontal velocity means it is at maximum height with $v_y = 0$ at time

$$v_y = \frac{\sqrt{2} v_0}{2} - g t = 0 \rightarrow t_c = \frac{\sqrt{2} v_0}{2g}.$$

To collide, both the target and the projectile must be at the same point at the time of the collision, meaning that $\vec{r}_P(t_c) = \vec{r}_T(t_c)$. Using the result found for the time of collision t_c , this means

$$\left[\frac{v_0^2}{2g} \right] \hat{i} + \left[\frac{v_0^2}{4g} \right] \hat{j} = [d] \hat{i} + \left[u \left(\frac{\sqrt{2} v_0}{2g} + T \right) - \frac{1}{2} g \left(\frac{\sqrt{2} v_0}{2g} + T \right)^2 \right] \hat{j}$$

Equating x-components, we obtain $v_0 = \sqrt{2gd}$.

(c) Using this result, and equating y-components, we have

$$u \left(\sqrt{\frac{d}{g}} + T \right) - \frac{1}{2} g \left(\sqrt{\frac{d}{g}} + T \right)^2 = \frac{d}{2}$$

Solving this quadratic equation for T , we obtain two roots

$$T + \sqrt{\frac{d}{g}} = \frac{u}{g} \pm \sqrt{\left(\frac{u}{g} \right)^2 - \frac{d}{g}} \rightarrow T = \frac{u}{g} \pm \sqrt{\left(\frac{u}{g} \right)^2 - \frac{d}{g}} - \sqrt{\frac{d}{g}}.$$

(d) For $u < \sqrt{gd}$, there is no real root, hence the scenario is not possible.

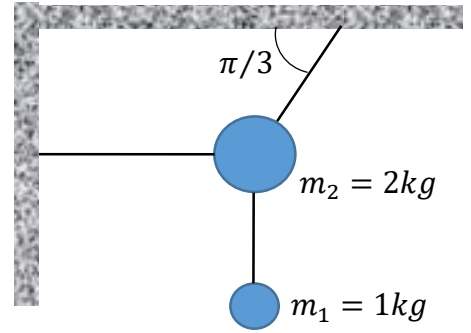
3. Two masses $m_1 = 1\text{ kg}$ and $m_2 = 2\text{ kg}$ are attached to each other and stationary walls by three inextensible ropes of negligible mass, as shown in the figure. Rope one is horizontal, rope two is vertical and rope three makes an angle $\pi/3$ with the horizontal as shown. Both masses can be treated as point particles. Gravitational acceleration is given as $g = 10\text{ m/s}^2$, and use $\sin \pi/3 = \sqrt{3}/2$.

a) (10 Pts.) Draw the free body diagram for both masses.

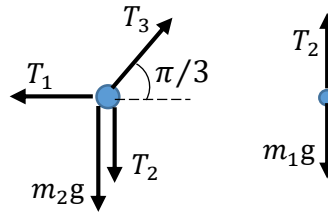
b) (10 Pts.) Find the tension all three ropes.

Now assume that the horizontal rope (rope 1) is cut:

c) (15 Pts.) Find the horizontal and vertical accelerations of both masses, right after the rope is cut. (Hint: Right after the horizontal rope is cut the remaining two ropes are still inextensible. ropes will not have time to move but their tension can change.)



Solution: (a)



(b) The system is in equilibrium. Therefore,

$$T_2 = m_1 g \rightarrow T_2 = 10\text{ N}.$$

$$T_3 \cos\left(\frac{\pi}{3}\right) = T_1 \rightarrow T_1 = \frac{T_3}{2}, \quad T_3 \sin\left(\frac{\pi}{3}\right) = T_2 + m_2 g \rightarrow \frac{\sqrt{3}}{2} T_3 = 30\text{ N} \rightarrow T_3 = 20\sqrt{3}\text{ N}$$

$$T_1 = \frac{T_3}{2} \rightarrow T_1 = 10\sqrt{3}\text{ N}.$$

(c) Right after the horizontal rope is cut $T_1 = 0$, and both masses will accelerate. According to Newton's second law

$$m_2 a_{2x} = T_3 \cos\left(\frac{\pi}{3}\right) = \frac{T_3}{2} \rightarrow a_{2x} = \frac{T_3}{4}$$

$$m_2 a_{2y} = m_2 g + T_2 - T_3 \sin\left(\frac{\pi}{3}\right) \rightarrow a_{2y} = 10 + \frac{T_2}{2} - \frac{\sqrt{3}}{4} T_3$$

$$a_{1x} = 0, \quad m_1 a_{1y} = m_1 g - T_2 \rightarrow a_{1y} = 10 - T_2$$

Since rope 2 is inextensible, $a_{2y} = a_{1y}$. This means

$$10 + \frac{T_2}{2} - \frac{\sqrt{3}}{4} T_3 = 10 - T_2 \rightarrow T_3 = 2\sqrt{3} T_2 \rightarrow a_{2y} = 10 - T_2, \quad a_{2x} = \frac{\sqrt{3}}{2} T_2$$

Rope 3 is also inextensible. This means the acceleration vector $\vec{a}_2$ is perpendicular to the direction of rope three, and, as can be seen in the figure $a_{2x}/a_{2y} = \tan(\pi/3) = \sqrt{3}$.

$$\frac{\sqrt{3}}{2} T_2 = 10\sqrt{3} - \sqrt{3} T_2 \rightarrow T_2 = \frac{20}{3}\text{ N}, \quad T_3 = \frac{40\sqrt{3}}{3}\text{ N}.$$

Therefore,

$$a_{1x} = 0, \quad a_{1y} = a_{2y} = \frac{10}{3}\text{ m/s}^2, \quad a_{2x} = \frac{10\sqrt{3}}{3}\text{ m/s}^2.$$

